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European Laboratory for Particle Physics

CTF3 Note 019 (Tech.)
(OTR, Thermal)

THERMAL ANALYSIS OF OTR SCREENS FOR CTF3

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Abstract

The Clic Test Facility stage 3 (CTF 3) will require emittance measurements in both the transverse and the longitudinal plane of the high intensity electrons drive beam.
The longitudinal measurement of the very short bunches (a few centimeters) is not trivial and only a few techniques are available. Among them the detection of Synchrotron Radiation, Optical Transition Radiation (OTR) and Cherenkov light. For all of the mentioned cases, the measurement is based on the conversion of photons into electrons (i.e. use of streak cameras).
The OTR method is the simplest to implement and gives very good time resolutions. One disadvantage of this technique is, however, that a radiator has to be inserted in the beam path, thus perturbing the beam. The energy deposited in the radiator by the impinging beam can also induce high temperatures, with the possibility of damaging the radiator itself.

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CTF-3 drive beam parameters

The CTF 3 drive beam at the end of the linac will have the characteristics described in Table 1.

Table 1: Beam parameters

Parameter	***Value***
Beam Current	3.5 A
Beam Size (σ)	0.25 ÷ 1.5 mm
Pulse Length	1.54 μs
Pulse Repetition Rate	5 ÷ 50 Hz
Beam Energy	184 ÷ 380 MeV

The charge density in a pulse can be rather high, up to ~8.5 10^{13} e^-/(mm^2 pulse), which leads to an important localized thermal load in any interposed radiator or screen, as a result of the interaction between the particles of the beam and the molecules of the radiator.

Thermal load analysis

Physical considerations

The electrons traversing the radiator loose energy by ionizing its molecules, with a process which is analog to the one described by the Bethe-Bloch formula, and by bremsstrahlung. If the target is sufficiently thin however, the X-rays are not reabsorbed in the radiator. For electrons of a few hundred MeV the bremsstrahlung represent a large fraction of the energy loss, 78% for electrons of 180 MeV in aluminum. The ionization dE/dx is almost independent of the energy of the incident particles for energies above 1 MeV and does not change sensibly from one material to another when normalized for the density , i.e. dE/dx expressed in [MeV cm^2/g]. Figure 3 shows the stopping power for electrons in aluminum as a function of the incident electrons energy.

Absorption of beam energy in the radiator

The assumption will be made that the energy lost by the beam due to bremsstrahlung is not reabsorbed by the radiator. This assumption is supported by the fact that the fraction of photons above 40 keV absorbed in 0.1 mm is negligible and assuming an X-ray spectrum like 1/E the integrated energy for photons below 40 keV is negligible as well.

Table 2: Parameters used in the calculation of absorption of photons in aluminum

μ_{en} = 0.36 cm^2/g	Photons absorption at 40 keV
ρ = 2.7 g/cm^3	Material density
δ = 100 μm	Target thickness

In fact the fraction of photons absorbed at 40 keV is:

$$\epsilon = 1 - e^{-\mu_{en}\rho\delta} = 0.01 \tag{1}$$

This means that only 1% of the photons in the 40 keV region are absorbed by the target, this can also be used as an upper limit for photons above 40 keV.

Assuming a spectrum like 1/E, photons below 40 keV represent only ~2 $10^{-4}$of the total X-Ray energy emission. Therefore we are left with an upper limit of 1% for the reabsorption which is already small enough to allow us to neglect it.
Another assumption will be that all the energy lost by ionization (collision) is entirely absorbed in the radiator. This means that the energy carried away by secondary emission particles is not considered.

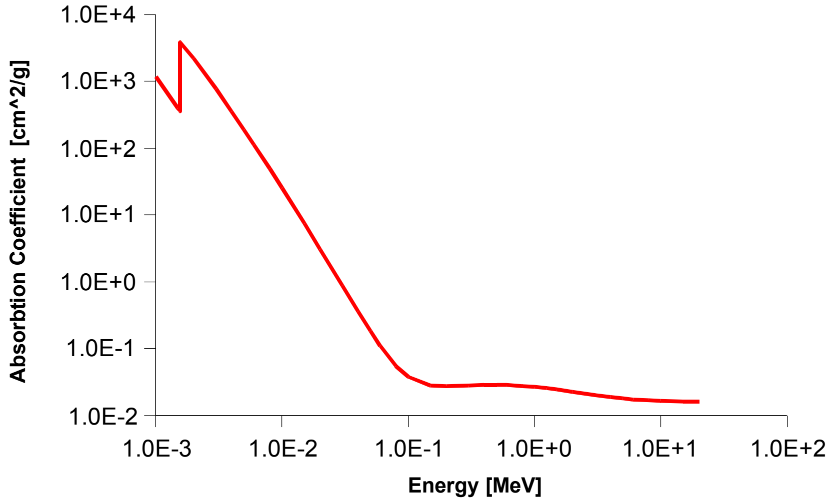


Fig. 1: Mass Energy Absorption Coefficient for X-Ray photons in Aluminum

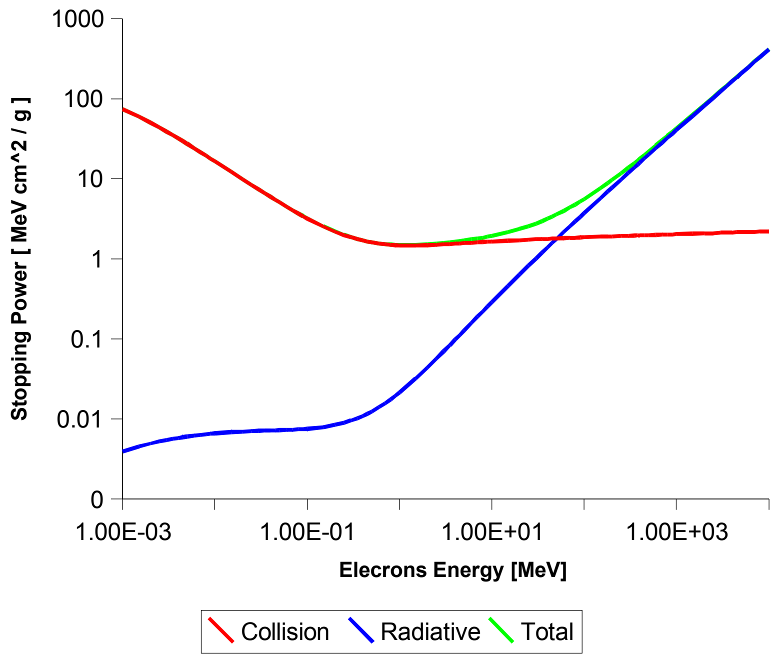


Figure 2: Stopping power for electrons in Aluminum

Black body radiation

The heat exchange due to black body (BB) radiation is quite small and can be neglected from the thermal analysis in order to to simplify it.
Considering graphite as an example, assuming a temperature of ~1500 °C and a beam spot of ~ 0.25 mm the black body emission is:

$$P = \epsilon \sigma T^4 A \tag{2}$$

Let us consider ε=1 (infrared):

$$P = 1 \cdot 5.67 \cdot 10^{-8} \cdot 1500^4 \pi (0.25 \cdot 10^{-3})^2 = 56 \cdot 10^{-3} \quad \mathrm{W} \tag{3}$$

The energy deposited on the target during one pulse can be expressed as:[1]

$$\Delta E = \frac{dE}{dx} \rho \delta I \Delta t = 2 \cdot 10^6 \cdot 2.3 \cdot 1 \cdot 10^{-2} \cdot 3.5 \cdot 1.5 \cdot 10^{-6} = 0.24 \quad \mathrm{J} \tag{4}$$

Comparing those two numbers one can see that the thermal emission, which is only interesting for about ten milliseconds after the pulse, represent only a small contribution to the total thermal balance of the target. This effect may make the cooling between pulses a little faster but it will be almost irrelevant on the maximum temperature attained by the radiator.
The blackbody radiation can however be a nuisance in the observation of the OTR photons. One should not neglect, however, that this radiation is emitted uniformly over 2π while OTR is emitted in a small cone. Wave lengths can also be different and opportune filters can help disentangling one from the other. As a last argument one should notice that while thermal emission is decaying slowly after the passage of the beam, compared to the length of the bunch, OTR consists of a very short pulse, of the order of the bunch length, so that background subtraction can be used.

Radiator temperature

The temperature variations with time can be computed using numerical methods. Table 2 describes the parameters of the radiator used in this analysis.

Table 3: Radiator Parameters

Parameter	***Value***
Thickness	100 μm
Outer radius	20 mm
Materials	Al, C, W, Sph., (Ti)

The profile of the ΔT induced by a short pulse, neglecting any heat exchange, can be expressed as:

$$\Delta T(r) = \frac{dE}{dx} \frac{N(r)}{c_p} \quad ; \quad \Delta T_{Gaussian}(r) = \frac{dE}{dx} \frac{N_{tot}}{2\pi \sigma^2 c_p} e^{-\frac{r^2}{2\sigma^2}} \tag{5}$$

Where N_{Tot} is the total number of particles in the pulse and dE/dx is in [J m^2/Kg]. This relation is only valid for a short pulse (~μs), otherwise one can not neglect the heat exchange (the time scale of heat exchange is of the order of milli seconds). The general relation can be expressed as:

$$\Delta T(r,t) = \frac{1}{c_p \rho \,\mathrm{dV}} \left[\frac{dE}{dx} N(r,t) \rho \,\mathrm{dV} + k \vec{\nabla} \cdot \vec{\nabla} T(r,t) \mathrm{dV} \right] \Delta t \tag{6}$$

Where $\vec{\nabla} \cdot \vec{\nabla} T(r,t)$ is the heat exchange rate, N(r, t) indicates the particle flux, i.e. the beam current divided by e^-, c_p and ρ are the specific heat and density of the material.

1 dE/dx in Mev cm^2 /g particle = J cm^2/g C therefore one can omit the 1.6 10^{-19} (MeV →□J) and use the beam current instead of the particle flux.

The term $\vec{\nabla}\cdot\vec{\nabla}\, T(r,t)\, dV$ can be rewritten as $\int \vec{\nabla}\, T\, \mathrm{dA}$ where the integral is extended to the surface of the volume dV .

The easiest way of calculating the temperature evolution is by numerical integration of eq. 6.

The steady state solution for the temperature profile is:

$$\Delta T(r) = \frac{\dot{Q}}{k} \frac{\ln \frac{R_e}{r}}{2\pi\delta} \tag{7}$$

Where $\dot{Q}$ is the total heat power, R_e is the outer radius of the radiator, δ is the thickness and k is the thermal conductibility. This formula is only valid for r > 5σ where steady state conduction can be assumed.

Numerical model

Let us assume that the beam and the radiator are both perfectly cylindersymmetric. We subdivide the radiator in a series of concentric rings and we compute the energy balance for each single ring in short time intervals, tacking into account the heat exchange between them and the energy deposited by the beam.

Consider ring *i* which is located between radii $r_i - \frac{\Delta r_i}{2}$ and $r_i + \frac{\Delta r_i}{2}$, the heat exchange with the adjacent inner ring will be:

$$\Delta Q_{i\leftarrow} = -k \frac{T_i - T_{i-1}}{r_i - r_{i-1}} 2\pi \left(r_i - \frac{\Delta r_i}{2}\right) \delta \Delta t \tag{8}$$

Where *k* is the heat conduction coefficient and δ is the thickness of the radiator.

We can write the same for the exchange with the adjacent outer ring:

$$\Delta Q_{i\rightarrow} = k \frac{T_{i+1} - T_i}{r_{i+1} - r_i} 2\pi \left(r_i + \frac{\Delta r_i}{2}\right) \delta \Delta t \tag{9}$$

The difference in sign keeps the quantity ΔQ positive for heat transfer toward ring *i*. In the case of a Gaussian beam the contribution of ionization to the energy balance is:

$$\Delta Q_{i\,\mathrm{beam}} = \frac{dE}{dx} \rho \delta \frac{I(r_i)}{2\pi\sigma^2 e}\ \mathrm{e}^{-\frac{r_i^2}{2\sigma^2}} 2\pi r_i \Delta r_i \Delta t \tag{10}$$

Where I is the total beam current, σ is the beam size and e is the electron charge.

This set of formulas need to be slightly modified for the first (inner-most) and last (outer-most) elements. In the case of the outer-most element we just force the heat-balance to zero with the assumption that a cooling system is keeping its temperature constant. For the inner most element eq. 8 has no meaning, eq. 9 becomes:

$$\Delta Q_{i\rightarrow} = k \frac{T_{i+1} - T_i}{r_{i+1}} 2\pi \Delta r_i \delta \Delta t \tag{11}$$

and eq. 10 becomes:

$$\Delta Q_{i\,\mathrm{beam}} = \frac{dE}{dx} \rho \delta \frac{I}{2\pi\sigma^2 e} \pi \Delta r_i^2 \Delta t \tag{12}$$

Figure 3 shows a schematics of the geometry used in the simulation.

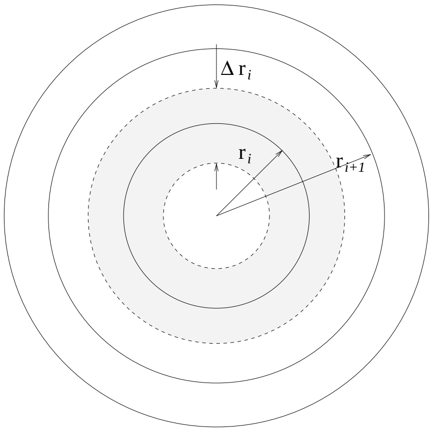


Fig. 3: Schematics of the geometry used in the model

Results

This section shows the results obtained for different radiator materials and different beam configurations.

Aluminum

Input parameters:

Beam Sigma	0.25 mm
Beam Current	3.5 A
Pulse Length	1.54 μs
Beam Energy	360 MeV
Repetition Rate	10 Hz
Analysis Length	50 Cycles
Melting Point	660 °C
Cp @ 273 K	0.84 J / g K
Cp @ 923 K	1.26 J / g K
K @ 273 K	235 W / m K

Results:

$\dot{Q}$	2.8 w
T_{max}	1650 °C (melting)
T_{calc}(r=3mm) (Eq.7)	56 °C
$T_{numeric}$(r=3mm)	48 °C

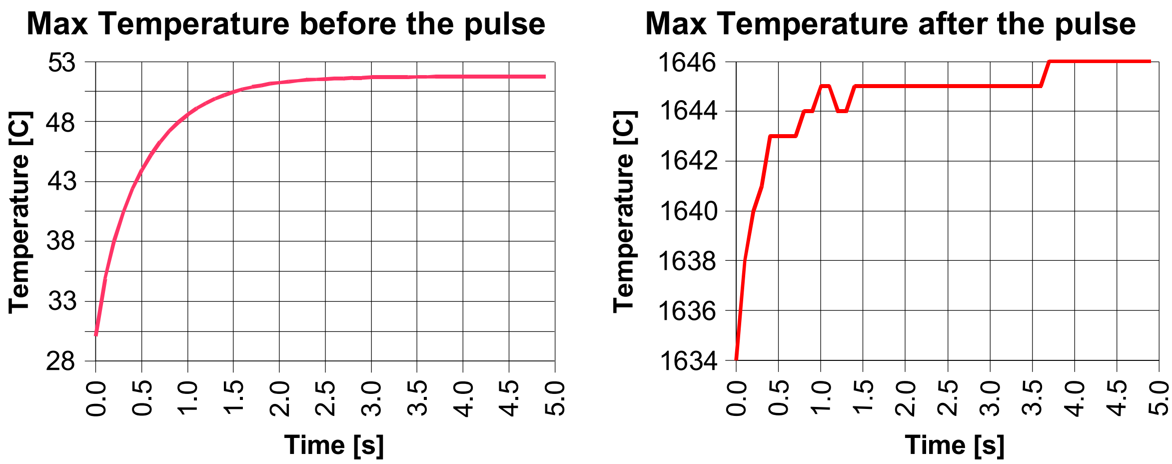


Fig. 4: Maximum temperature at different points in the cycle

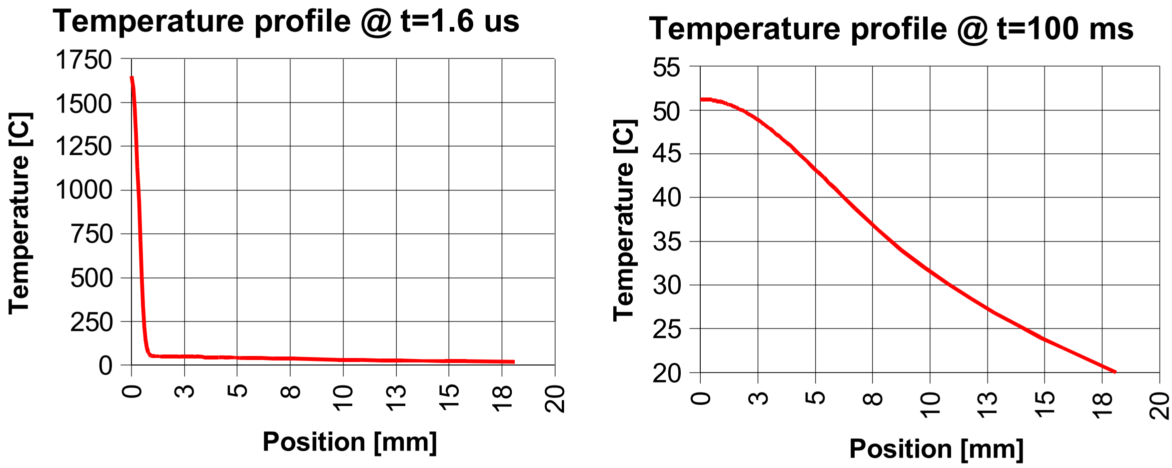


Fig. 5: Temperature profiles at different points in the cycle

Input parameters:

Beam Sigma	0.4 mm
Beam Current	3.5 A
Pulse Length	1.54 µs
Beam Energy	360 MeV
Repetition Rate	10 Hz
Analysis Length	50 Cycles

Results:

$\dot{Q}$	2.8 w
T_{max}	930 °C (melting)
T_{calc}(r=3mm) (Eq.7)	56 °C
$T_{numeric}$(r=3mm)	48 °C

Input parameters:

Beam Sigma	0.5	mm
Beam Current	3.5	A
Pulse Length	1.54	µs
Beam Energy	360	MeV
Repetition Rate	10	Hz
Analysis Length	50	Cycles

Results:

$\dot{Q}$	2.8	w
T_{max}	680	°C (melting)
T_{calc}(r=3mm) (Eq.7)	56	°C
$T_{numeric}$(r=3mm)	48	°C

Input parameters:

Beam Sigma	0.6	mm
Beam Current	3.5	A
Pulse Length	1.54	µs
Beam Energy	360	MeV
Repetition Rate	10	Hz
Analysis Length	50	Cycles

Results:

$\dot{Q}$	2.8	w
T_{max}	510	°C
T_{calc}(r=3mm) (Eq.7)	56	°C
$T_{numeric}$(r=3mm)	48	°C

Input parameters:

Beam Sigma	0.6	mm
Beam Current	3.5	A
Pulse Length	1.54	µs
Beam Energy	360	MeV
Repetition Rate	50	Hz
Analysis Length	50	Cycles

Results:

$\dot{Q}$	14	w
T_{max}	650	°C
T_{calc}(r=3mm) (Eq.7)	200	°C
$T_{numeric}$(r=3mm)	170	°C

Graphite

Input parameters:

Beam Sigma	0.25 mm
Beam Current	3.5 A
Pulse Length	1.54 µs
Beam Energy	360 MeV
Repetition Rate	10 Hz
Analysis Length	50 Cycles
Sublimation Point	3530 °C
Cp @ 273 K	0.644 J / g K
Cp @ 1866 K	1.96 J / g K
K @ 273 K	157 W / m K [2]

Results:

$\dot{Q}$	2.5 w
T_{max}	1730 °C
T_{calc}(r=3mm) (Eq.7)	68 °C
$T_{numeric}$(r=3mm)	61 °C

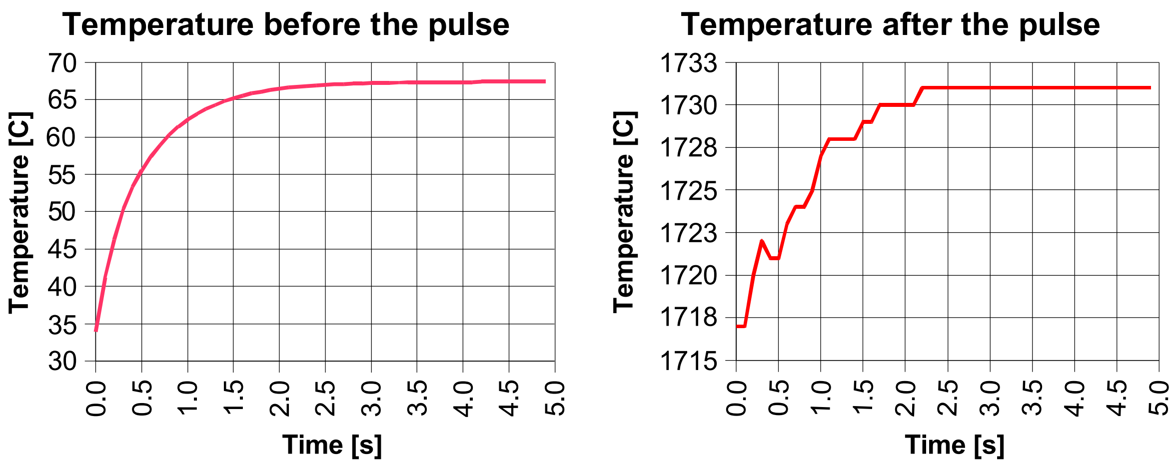
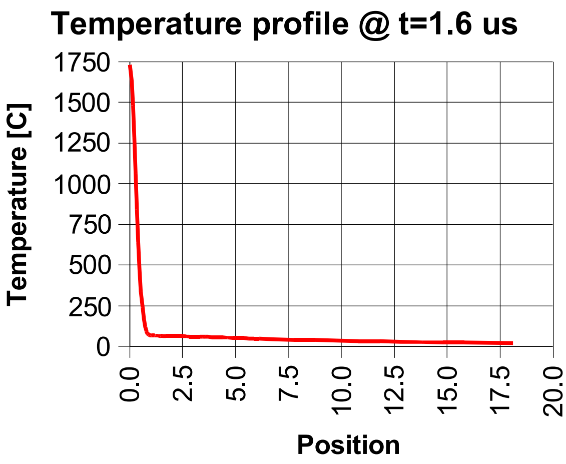


Fig. 6: Maximum temperature at different points in the cycle

2 Many different kinds of graphite exists with extremely broad ranges for Cp and K.



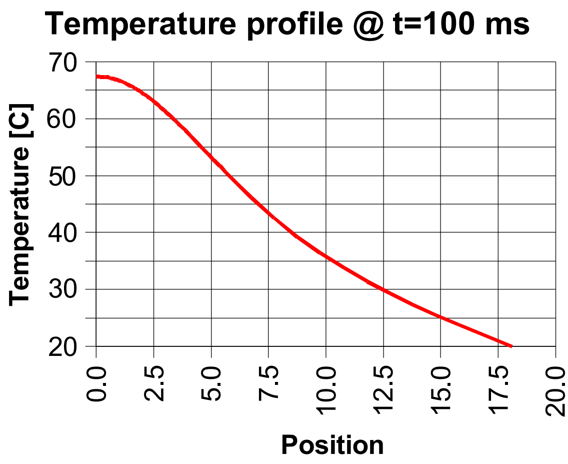


Fig. 7: Temperature profiles at different points in the cycle

Input parameters:

Beam Sigma	0.25 mm
Beam Current	3.5 A
Pulse Length	1.54 µs
Beam Energy	360 MeV
Repetition Rate	50 Hz
Analysis Length	50 Cycles

Results:

$\dot{Q}$	12.5 w
T_{max}	2250[3] °C
T_{calc}(r=3mm) (Eq.7)	260 °C
$T_{numeric}$(r=3mm)	300 °C

Tungsten

Input parameters:

Beam Sigma	0.25 mm
Beam Current	3.5 A
Pulse Length	1.54 µs
Beam Energy	360 MeV
Repetition Rate	10 Hz
Analysis Length	50 Cycles
Melting Point	3422 °C
Cp @ 273 K	0.133 J / g K
Cp @ 3100 K	0.2 J / g K
K @ 273 K	170 W / m K

3 Tacking into account the blackbody emission this number will be 2125 °C which shows how little black body radiation contributes to the thermal balance of the radiator

Results:

$\dot{Q}$	15.6 w
T_{max}	8715 °C (melting)
T_{calc}(r=3mm) (Eq.7)	300 °C
$T_{numeric}$(r=3mm)	266 °C

Summary

Table 4 shows a summary of the results of the numeric computations. It is clear how small beams below 0.6 mm sigma will be very dangerous for the existing installations (actual OTR's are made of aluminum and cherenkov radiators are made of sapphire).
Another important result is that graphite can stand the challenge. Graphite has been already used in similar applications (OTR and stripper foils). The drawback of graphite is that it is not a reflective material which makes the observation of reflected transition radiation almost impossible, although the forward emission is not affected by the reflectivity of the surface.
All calculations have been performed for a foil of 100 μm thickness. By reducing considerably this parameter one can reduce the heat deposition in the radiator (linear relation). This does not influence at all the temperature jump during the beam pulse since the time scale is too short for any conduction or BBR to affect it and the only key parameter here is the specific heat of the material. The thickness can however make a difference for the maximum temperature, especially for material with small thermal conductibility or for higher repetition rates, by making the BB radiation more effective (~independent from the thickness) and thus augmenting the cooling-down in-between pulses.

I=3.5 A t=1.54 μs E=360 MeV		
Aluminum Cp_{273} = 0.84 J/g K K_{273} = 235 w/m K T_{melt} = 660 °C		
σ	**10 Hz**	**50 Hz**
0.25 mm	1650 °C	--
0.40 mm	930 °C	--
0.50 mm	680 °C	--
0.60 mm	510 °C	650 °C
Graphite Cp_{273} = 0.64 J/g K K_{273} = 157 w/m K T_{subl} = 3530 °C		
0.25 mm	1730 °C	2250 °C
Tungsten Cp_{273} = 0.13 J/g K K_{273} = 170 w/m K T_{melt} = 3422 °C		
0.25 mm	8715 °C	--
0.40 mm	4600 °C	--
0.50 mm	3340 °C	4560 °C
0.60 mm	2550 °C	3880 °C
Sapphire Cp_{273} = 0.75 J/g K K_{273} = 41 w/m K T_{soft} = 1800 °C		
0.25 mm	3850 °C	
0.40 mm	1750 °C	3500 °C
0.50 mm		2975 °C

Table 4 Summary of temperature variations for different situations

Photon emission

Black Body radiation

The black body radiation is governed by the well known Planck equation:

$$E(\lambda, T) = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{\frac{hc}{kT\lambda}} - 1} \tag{13}$$

Here E(λ, T) is the energy density per unit wavelength at wavelength λ and temperature T in the black body cavity, h is the Planck constant, c is the speed of light and k is the Boltzman constant. To get the photon density at a given wavelength in the cavity one has to divide eq. 13 by hc/λ which becomes:

$$N(\lambda, T) = \frac{8\pi}{\lambda^4} \frac{1}{e^{\frac{hc}{kT\lambda}} - 1} \tag{14}$$

The rate of photons emitted per unit area by a hole in the black body cavity is given by multiplying eq. 14 by c/4:[4]

$$I(\lambda, T) = \frac{2\pi c}{\lambda^4} \frac{1}{e^{\frac{hc}{kT\lambda}} - 1} \tag{15}$$

Table 5 shows the number of photons emitted per square meter in certain wavelength ranges of interest and different temperatures. These numbers have been calculated for a BB so one has to multiply by the surface emissivity of the particular material, this can vary from a few percent to hundred percent.

λ [nm]	**500 °C**	**800 °C**	**1000 °C**	**1600 °C**	**2000 °C**
100÷200	1.25E-13	3.00E-02	1.42E+03	1.48E+11	1.54E+14
200÷300	1.21E+00	5.93E+07	8.03E+10	2.15E+16	2.42E+18
300÷400	3.90E+06	2.49E+12	5.81E+14	7.59E+18	2.76E+20
400÷500	2.83E+10	1.33E+15	1.08E+17	2.27E+20	4.10E+21
500÷600	1.04E+13	8.44E+16	3.34E+18	1.98E+21	2.20E+22
600÷700	6.35E+14	1.47E+18	3.45E+19	8.19E+21	6.40E+22
700÷800	1.32E+16	1.16E+19	1.84E+20	2.18E+22	1.31E+23
800÷900	1.32E+17	5.48E+19	6.36E+20	4.37E+22	2.13E+23
900÷1000	7.83E+17	5.48E+19	1.60E+21	7.13E+22	2.95E+23

Table 5 Surface emission of a black body in photons/(m^2 s) for different temperatures

4 The term c is used to go from photon density to photon flux and ½ is because we only want to consider the photons which are leaving the cavity. I don't know the reason for the remaining ½ but it is needed in order to agree with the Stephan Boltzman formula and is found in literature.

Optical Transition Radiation

The emission of electromagnetic radiation by a particle crossing the boundary between two different materials (in the approximation of perfect conductor for at least one of them) is:[5]

$$\frac{dW}{d\Omega\, d\omega}=\frac{e^2}{c\pi^2\epsilon_0}\frac{\beta^2\sin^2\theta}{[1-\beta^2\cos^2\theta]^2} \tag{16}$$

Which for an energy of 360 MeV gives the following normalized spectra (integrated over 4π):
The cut-off frequency should be around 50 nm (calculated using ω_p=3.81 10^{16} which *should be* the value for carbon). The emission angle is $2/\gamma$,this is valid for a single particle. In the case of a particle beam the resulting radiation is the convolution of the emission of each individual particle in the beam. The consequence is that the divergence of the beam contributes to the total aperture of the emitted radiation. For high energy beams, i.e. small OTR emission angle, and strong focusing, i.e. large beam divergence, this effect can be important.

Wavelength [nm]	**Photons / e^-**
100 ÷ 200	2.49E-02
200 ÷ 300	1.46E-02
300 ÷ 400	1.03E-02
400 ÷ 500	8.02E-03
500 ÷ 600	6.56E-03
600 ÷ 700	5.54E-03
700 ÷ 800	4.80E-03
800 ÷ 900	4.24E-03
900 ÷ 1000	3.79E-03

Signal to noise

Let us consider the case of graphite at 2000 °C for a device with an half-aperture of 10°:

Black Body emission (300 ÷ 900 nm)	1.4 10^{18} photons/s (r= 1mm)
Black Body acceptance	1.50%
OTR (300 ÷ 900 nm)	0.039 photons/e^-
OTR acceptance (no beam divergence)	60.00%
Bunch length (4σ)	33 ps
Black Body photons detected	0.675 10^6
OTR photons detected (1.2 nC/bunch)	175 10^6
Signal/Noise	260

From this example it is evident that black body radiation is not a concern as a source of background light.

5 The term ε_0 doesn't appear in the papers and books I have consulted, however the introduction of ε_0 puts the dimensional analysis straight and gives better agreement with numbers mentioned in other works. It is probably the case that the formula found in literature has been derived in the CGS electromagnetic system where ε_0=1 (this was however never mentioned). In CLIC Note 211 (PS 93 - 40 BD) S. Battisti gives a formula including the ε_0 term, however the ε_0 seems to be the only thing right in that set of formulas!

Conclusions

The high charge of the CTF3 drive beam together with it's small size poses serious problems to the use of intercepting radiators for beam diagnostics. The problem of the thermal heating due to ionization in the radiators as been investigated with the result that none of the existing equipment can be used in the extreme conditions foreseen for CTF3. It is possible however to use the existing diagnostic devices in the normal CTF3 operation (larger beam size). A possible improvement might come from the introduction of graphite radiators. Graphite is unfortunately not a very reflective material which makes the observation of the reflected OTR very inefficient (< 20%). The forward emission can be used but this poses serious problems on the observation techniques (holed mirrors, bending magnets etc.).
<u>This computation assumes a cooled radiator which may not be the case for the existing installations</u>. The mechanical support of the radiators, though, may be already sufficient for this purpose (seen the small power involved < 10 w).
It was also showed that BB radiation is not a nuisance in the observation of OTR even in the case of high temperature radiators.
Titanium has not been included in the investigated materials because I didn't have sufficient information about its thermo-mechanical behavior. Titanium has a high melting point, 1953 K, but has a phase transition at 1156 K. This phase transition helps absorbing the deposited energy but very fast and localized repeated phase transitions may damage the radiator or change the properties of the material
The value of Cp for Ti is 0.5 [J / g K], a little lower than aluminum, which means that the top temperature will be higher than the transition temperature (even above melting) and dealing with the behavior in the phase transition is not straight forward.